Written Exam for the M.Sc. in Economics 2012-II

Advanced Industrial Organization

Final Exam

8 June, 2012

(3hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

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ALL QUESTIONS BELOW SHOULD BE ANSWERED

1. **Problem 1.**

   We consider a Cournot oligopoly with n firms producing a homogeneous output.

   The firms are symmetric and all have the cost function

   $$C(x_i) = cx_i,$$

   where c is a constant and x_i is the production of firm i.

   The price, p, is determined by the inverse demand function

   $$p = a - bX$$

   where $X = \sum_{i=1}^{n} x_i$ is total production and $a, b > 0$.

   (a) Find the symmetric Cournot equilibrium, and expressions for production per firm x, price, p and profit per firm, π.
   Let X_{-i} denote the total production of the other firms than i.
   Profit of i is

   $$(a - b(X_{-i} + x_i))\, x_i - cx_i$$

   foc (Unfortunately, many just insert $X_{-i} = (n-1)\, x_i$ before taking the first order condition. This is not valid, in this way, you change the production of everybody when you contemplate the effects of a small change in x_i.). One should take X_{-i} as given

   $$\frac{\partial\left(\left(a - b\left(X_{-i} + x_i\right)\right) x_i - cx_i\right)}{\partial x_i} = 0$$

   so that the reaction of i is

   $$x_i = \frac{1}{2b}\left(a - c - bX_{-i}\right) \qquad \text{(react fctn)}$$

   Now use that in sym eq $X_{-i} = (n-1)\, x_i$, which gives

   $$x_i = \frac{1}{2b}\left(a - c - b\left(n-1\right) x_i\right)$$

so that

$$x_i = x = \frac{a-c}{b+bn}$$

hence

$$p = a - bn\frac{a-c}{b+bn} = \frac{a+cn}{n+1}$$

and

$$\begin{aligned} \pi &= \left(\frac{a+cn}{n+1}\right)\frac{a-c}{b+bn} - c\frac{a-c}{b+bn} \\ &= \frac{(a-c)^2}{b(n+1)^2} \end{aligned}$$

(b) Now consider a merger among two firms. Assume that the merger gives rise to synergies, so that the merged firm's cost function is

$$C_m(x_m) = mx_m$$

where $m \leq c$.

Find the Cournot equilibrium after the merger. (Remember that the equilibrium is *not* symmetric after the merger). You should find expressions for the production of the merged firm x_m, the production per firm of the non-merged firms, x, the price p and the profit to the merged firm π_m .

Reaction function of merged firm (just insert in $(reactfctn)$ above, and let X_{-m} denote the total production of the other firms)

$$x_m = \frac{1}{2b}(a - m - bX_{-m})$$

In equilibrium, where the other firms choose the same quantity, x

$$X_{-m} = (n-2)x$$

so we can write reaction fctn of merged firm

$$x_m = \frac{1}{2b}(a - m - b(n-2)x)$$

and that of the non-merged firms (again just insert in $(reactfctn)$ above))

$$x = \frac{1}{2b}(a - c - b((n-3)x + x_m))$$

which gives

$$x = \frac{1}{bn}\left(a - 2c + m\right), x_m = \frac{1}{bn}\left(a - 2c + m + cn - mn\right)$$

So price is

$$p = a - b\left(\left(n-2\right)\frac{1}{bn}\left(a - 2c + m\right) + \frac{1}{bn}\left(a - 2c + m + cn - mn\right)\right)$$

which gives

$$p = \frac{a - 2c + m + cn}{n}$$

and

$$\begin{aligned}
\pi_m &= px_m - m\left(x_m\right) \\
\pi_m &= \frac{a - 2c + m + cn}{n}\frac{1}{bn}\left(a - 2c + m + cn - mn\right) - m\frac{1}{bn}\left(a - 2c + m + cn - mn\right) \\
&= \frac{\left(a - 2c + m + cn - mn\right)^2}{bn^2}
\end{aligned}$$

(c) Show that the merger leads to a lower price if and only if the merger gives a sufficiently large synergy, namely if and only if

$$c - m > \frac{a - c}{n + 1} \tag{1}$$

The price has decreased as a result of the merger if

$$\begin{aligned}
\frac{a - 2c + m + cn}{n} - \frac{a + cn}{n + 1} &< 0 \\
\frac{a - 2c + m - cn + mn}{n\left(n + 1\right)} &< 0 \\
a - c + \left(m - c\right)\left(n + 1\right) &< 0 \\
\frac{a - c}{n + 1} &< c - m
\end{aligned}$$

Discus this result in view of the general results you know about mergers which does not lead to synergies from Farrell and Shapiro.

Farrell and Shapiro shows that a merger which does not lead to a synergy leads to increasing prices. This is in accordance with our result here. In fact, our result shows exactly how large the synergy should be in the linear example .

(d) As a rule of thumb one may say, that if the involved firms have a sufficiently small joint market share a competition policy authority is likely to clear the merger. Does this make sense in view of condition (1).

Yes, the condition is related to the number of fims. The more firms (and therefore the smaller market share each firm has, the easier is the condition to fulfill).

(e) Show that the merger is profitable for the merging firms if and only if

$$c - m > \frac{\left(1+\sqrt{2}\right) n + 1}{n+1} \frac{a-c}{n+1} \qquad \text{(wrong)}$$

Well unfortunately there were three sign errors in the expression. The right condition is

$$c - m > \frac{\left(\sqrt{2}-1\right) n - 1}{n-1} \frac{a-c}{n+1} \qquad \text{(correct)}$$

To see this, proceed as follows: The merger is profitable if

$$\pi_m > 2\pi_{premerger}$$

$$\begin{aligned}
\frac{(a-2c+m+cn-mn)^2}{bn^2} &> 2\frac{(a-c)^2}{b(n+1)^2} \\
\frac{(a-c+(c-m)(n-1))^2}{n^2} &> 2\frac{(a-c)^2}{(n+1)^2} \\
\frac{(a-c+(c-m)(n-1))}{n} &> \sqrt{2}\frac{(a-c)}{(n+1)} \\
((c-m)(n-1)) &> \sqrt{2}\frac{(a-c)}{(n+1)}n - \frac{(a-c)(n+1)}{(n+1)} \\
((c-m)(n-1)) &> \sqrt{2}\frac{(a-c)}{(n+1)}n - \frac{(a-c)(n+1)}{(n+1)} \\
((c-m)(n-1)) &> \left(\frac{\sqrt{2}}{(n+1)}n - \frac{(n+1)}{(n+1)}\right)(a-c) \\
c-m &> \frac{\left(\sqrt{2}-1\right)n-1}{n-1}\frac{a-c}{n+1}
\end{aligned}$$

As $\frac{(\sqrt{2}-1)^{n-1}}{n-1} < 1$, *this condition is more slack than the one derived in c.*

However, the condition given in the exercise (wrong) is more strict as $\frac{(1+\sqrt{2})^{n+1}}{n+1} > 1$.

Given the fact that the condition given in the exercise is wrong, any answer which has struggled in the right direction here is considered as ok. Only a few tried seriously to answer the question. They have all got a "correct" in the marking here.

(f) Suppose that the competition authority knows all details about the market and the firms except that it does not know the synergy effects of the merger, i.e. the competition authority does not know the value of m. Suppose further that the competition authority is interested in consumer surplus. Should the competition authority clear the merger if the firms ask for it?

The correct answer should compare the conditions given in c and d. If the student rely on the (wrong) condition given in the text, then the answer is taht since this is more strict, the merger can safely be cleared. The firms only ask for a merger if it is profitable and if the condition in d is more strict than c this implies taht the price will fall. However, under the rightly derived condition given here, this is not correct and the authority has to make a further investigation, and clear the merger if it finds that the condition in c is met.

(g) Without deriving the result explain in words the central result in Farrell and Shapiro concerning the welfare effects of mergers in Cournot markets. Discuss the problems with implementing the criterion for a welfare improving merger empirically.

See Farrell and Shapiro.

(h) Farrell and Shapiro consider mergers in a static Cournot model. In competition authority lingo, such effects are called one-sided effects. When a competition authority considers a merger it should investigate one-sided as well as so-called coordinated effects, which are effects pertaining to tacit collusion and cartel behavior. Explain and discuss the so-called Airtour's conditions which competition authorities use to evaluate coordinated effects of mergers.

Explain - in words no formulas - how they are related to the theoretical contributions of Stigler and Green and Porter.

See discussion of LM merger in slides and Stigler, Green-Porter. (also Motta chap 5)

2. Two firms produce a homogenous product and compete in prices. The firm setting the lowest price takes the whole market. If the firms set the same price, they split the market equally. The firms have marginal costs equal to 1. There is an infinite number of periods: $t = 0, 1, \ldots, \infty$. The per-period discount rate is $\delta < 1$. There is a continuum of consumers of size 1. The consumers have unit demand, and the willingness to pay for the good is v_t in period t. The willingness to pay is deterministic but cyclical: $v_0 = 3$, $v_1 = 2$, $v_2 = 3$, $v_3 = 2$, $v_4 = 3\ldots$ The firms are aware of this.

(a) Derive the equation that determines the critical discount factor $\underline{\delta}$ above which the firms can sustain tacit collusion on the monopoly price in all periods. Show that $\underline{\delta} > ½$.

Answer: *The monopoly price in period t is v_t. Assuming that the firms play grim trigger strategies, collusion on the monopoly prices can be sustained if the following two conditions, which ensure that there is not an incentive to deviate, hold:*

$$\frac{3-1}{2}+\frac{\delta(2-1)}{2}+\frac{\delta^2(3-1)}{2}+\cdots=\frac{2}{2(1-\delta^2)}+\frac{\delta}{2(1-\delta^2)}=\frac{2+\delta}{2(1-\delta^2)}\geq 2 \Leftrightarrow \frac{1}{2(1-\delta^2)}\geq\frac{2}{2+\delta} \text{ (High demand)}$$

$$\frac{2-1}{2}+\frac{\delta(3-1)}{2}+\frac{\delta^2(2-1)}{2}+\cdots=\frac{1}{2(1-\delta^2)}+\frac{\delta}{1-\delta^2}=\frac{1+2\delta}{2(1-\delta^2)}\geq 1 \Leftrightarrow \frac{1}{2(1-\delta^2)}\geq\frac{1}{1+2\delta} \text{ (Low demand)}$$

Since $\frac{2}{2+\delta} > \frac{1}{1+2\delta}$, the incentive to deviate is strongest in odd periods where the willingness to pay is high. Hence, $\underline{\delta}$ is defined implicitly as the solution to $\frac{1}{2(1-\delta^2)} = \frac{2}{2+\delta}$. For $\delta = ½$, we have that $\frac{1}{2(1-(1/2)^2)} = \frac{2}{3} < \frac{2}{2+1/2} = \frac{4}{5}$, so $\underline{\delta} > \frac{1}{2}$.

Suppose in the following two questions that $\delta < \underline{\delta}$ such that collusion on the monopoly price every period is not possible.

(b) Show that the firms are able to sustain tacit collusion for $½ \leq \delta < \underline{\delta}$. Derive the optimal collusive prices in odd and even periods.

Answer: *The critical issue is to avoid deviation in the odd periods where the high willingness to pay is high. Arguing as Rotemberg and Saloner (1986), the price in even periods should be set equal to the monopoly price 2 in order to facilitate collusion. This minimizes the incentive to deviate in odd periods, because the value of future collusion is maximized. Hence, the highest price*

in odd periods that still allows collusion to be sustained is given as the solution to the following equation:

$$\frac{p_{odd}-1}{2}+\frac{\delta(2-1)}{2}+\frac{\delta^2(p_{odd}-1)}{2}+\cdots=\frac{p_{odd}-1}{2(1-\delta^2)}+\frac{\delta}{2(1-\delta^2)}=\frac{p_{odd}-1+\delta}{2(1-\delta^2)}=p_{odd}-1.$$

It can be verified that for $\delta = ½$ *collusion can be sustained by setting the price in all periods equal to 2. Hence, the firms can sustain collusion for* $\delta \geq ½$ *by making the difference in profits between odd and even periods sufficiently small.*

(c) Suppose now instead that the willingness to pay is not deterministic, still either 2 or 3, but that the value in period *t +1* is the same as in period *t* with probability *p* where $p > ½$. Discuss – in words, no formulas – how this affects the scope for tacit collusion in the industry considered.

Answer: *It is here not specified whether the demand shocks are observable (as in Rotemberg and Saloner (1986)) or unobservable (as in Green and Porter (1984)). In the following, observable shocks are assumed, but an answer that assumes unobservable shocks and draws on Green and Porter is also fine. Correlation between the willingness to pay across time facilitates collusion. To see this, consider a period with high willingness to pay. The profit from deviating is the same compared to the model considered above, or one of uncorrelated willingness to pay across time as in Rotemberg and Saloner (1986), but the future profits from colluding is higher as the net present value of the consumers' willingness to pay is higher.*

3. The market for handheld game consoles first took off with the appearance of Nintendo's *Game Boy* in 1989, the first device to sell to the mass market. Handheld game consoles are part of a system comprising both hard- and software. Hardware manufacturers supply consoles and often also software titles, while software providers concentrate on the development and distribution of games. Hardware suppliers actively manage the quality of the market's software side: developers need to sign detailed licensing contracts which are then enforced by legal and technological means such as security chips. This also prevents any hardware manufacturer from developing consoles that are compatible with games for other platforms.

Industry observers typically separate consoles into generations. In industry terminology, generations IV to VII are considered here. Table 1 provides an overview of the consoles in the different generations in terms of specifications, launch date, etc. "Backward Compatibility" refers to whether games developed for the previous generation console from the same company can be played on a given console. Figure 1 illustrates the market shares of the different consoles.

(a) Explain why consumers might care about the number of other consumers that have bought the same type of console.

Answer: *This is a market characterized by network externalities. The more users buy a certain game console, the more games will be developed for the game console, and the higher is the utility from playing on the game console.*

(b) Drawing on the theories covered in class, try to explain the evolution of the market shares observed in the market for handheld game consoles. In particular, think about the role that backward compatibility plays in this market.

Answer: *An answer could include some of the following considerations:*

i) *In generation IV, Nintendo had the lightest and most powerful game console, which gave them the lead in the market.*

ii) By making the later generations backwards compatible (with the exception of Virtual Boy), Nintendo was able to leverage this lead by making it possible to play the many games developed for the older generations on the newest game console. This allowed Nintendo to become the market leader also in generation VI where it did not have the game console with the best specifications.

iii) The only non-Nintendo game console that has been able to obtain a significant market share since 1999 is Playstation Portable, which had much better technical specifications than the competing Nintendo game console.

Table 1: Mobile handheld consoles sold between 1995 and 2007

Console	Platform	Backward Compatibility	U.S. launch	Manufacturer	Hardware	
					CPU [MHz]	Weight [g]
Generation IV						
Game Boy	Game Boy	No	8/1989	Nintendo	4.2	300
Game Boy Pocket			9/1996			148
Game Gear	Game Gear	No	1/1991	Sega	3.6	500
Generation V						
Game Boy Color	GB Color	Yes	11/1998	Nintendo	8.4	188
Virtual Boy	Virtual Boy	No	8/1995	Nintendo	20	760
game.com	game.com	No	9/1997	Tiger	10	380
game.com Pocket Pro			12/1999			n/a
Generation VI						
Game Boy Advance	Game Boy Advance	Yes	6/2001	Nintendo	16.7	180
Game Boy Advance SP			3/2003			142
Neo Geo Pocket Color	NGP Color	No	8/1999	SNK	6.14	145
N-Gage	N-Gage	No	10/2003	Nokia	104	137
N-Gage QD			8/2004			143
Generation VII						
DS	DS	Yes	11/2004	Nintendo	67	275
DS Lite			6/2006			218
Playstation Portable	Playstation Portable	No	3/2005	Sony	333	280
Playstation Portable Slim			9/2007			189

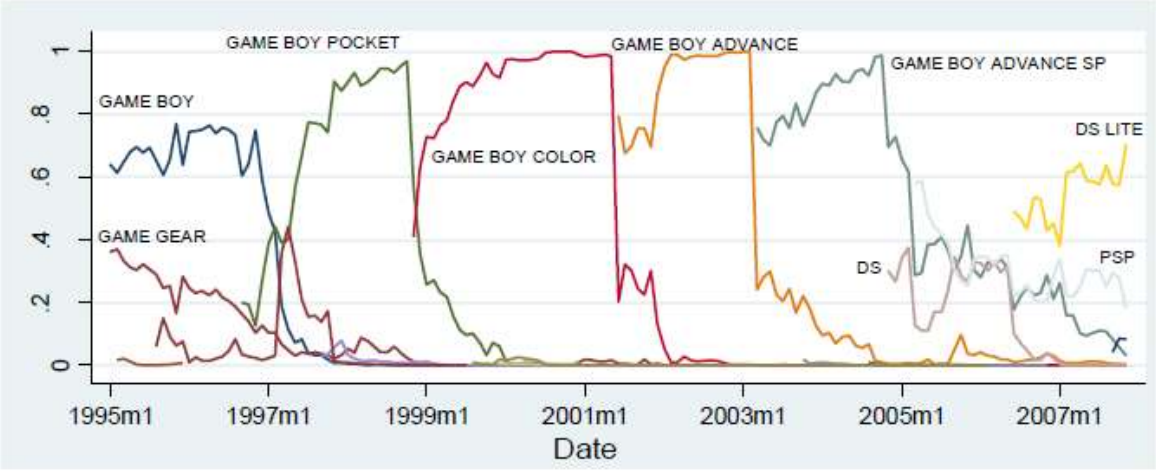

Figure 1: Monthly market shares from 1995 to 2007